Version

General Certificate of Education (A-level)
January 2013

Mathematics MM2B

(Specification 6360)

Mechanics 2B

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	KE = $\frac{1}{2} \times 0.16 \times 11^2$	M1		
	= 9.68 J	A1	2	
(b)	Change in PE: mgh = 0.16 × 9.8 × 5	M1		
	= 7.84 J	A1	2	
(c)(i)	KE when reached point B			
	= 9.68 – 7.84 J	M1		**'(a)' – '(b)'**
	= 1.84 J	A1	2	cao
(ii)	Speed of ball is $\sqrt{\dfrac{1.84}{\frac{1}{2} \times 0.16}}$	M1		If added in **(c)(i)** 0 marks for **(c)(i)** 14.8 M1A1for **c(ii)**
	= 4.7958 m s^{-1}			
	= 4.80 m s^{-1}	A1	2	Condone 4.8,4.79
	Total		**8**	
2(a)	$\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$	M1		
	$= -4\pi \sin\left(\dfrac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}$	A1	2	M1 for either term correct Accept $-12 \times \dfrac{\pi}{3}\sin\left(\dfrac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}$ condone no **i** in **(a)**
(b)(i)	Using **F** = m**a**:			
	$\mathbf{F} = 4 \times \left[-4\pi \sin\left(\dfrac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}\right]$	M1		Or either term correct
	$\mathbf{F} = -16\pi \sin\left(\dfrac{\pi}{3}t\right)\mathbf{i} - 72t\mathbf{j}$	A1	2	
(ii)	When $t = 3$, $\mathbf{F} = 4 \times [-4\pi \sin(\pi)\mathbf{i} - 54\mathbf{j}]$			
	$= -216\mathbf{j}$	B1		
	Magnitude is 216	B1ft	2	ft finding magnitude of their F
(c)	$\mathbf{r} = \int \mathbf{v}\, dt$	M1		either term correct
	$= \dfrac{36}{\pi}\sin\left(\dfrac{\pi}{3}t\right)\mathbf{i} - 3t^3\mathbf{j} + \mathbf{c}$	A1		No need for **c** (otherwise cao) Condone $\dfrac{12}{\left(\pi/3\right)}$
	When $t = 3$, $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j}$	M1		
	$\rightarrow\ -81\mathbf{j} + \mathbf{c} = 4\mathbf{i} - 2\mathbf{j}$			
	$\mathbf{c} = 4\mathbf{i} + 79\mathbf{j}$	A1		
	$\mathbf{r} = \left\{\dfrac{36}{\pi}\sin\left(\dfrac{\pi}{3}t\right) + 4\right\}\mathbf{i} + \{79 - 3t^3\}\mathbf{j}$	A1	5	cao
	Total		**11**	

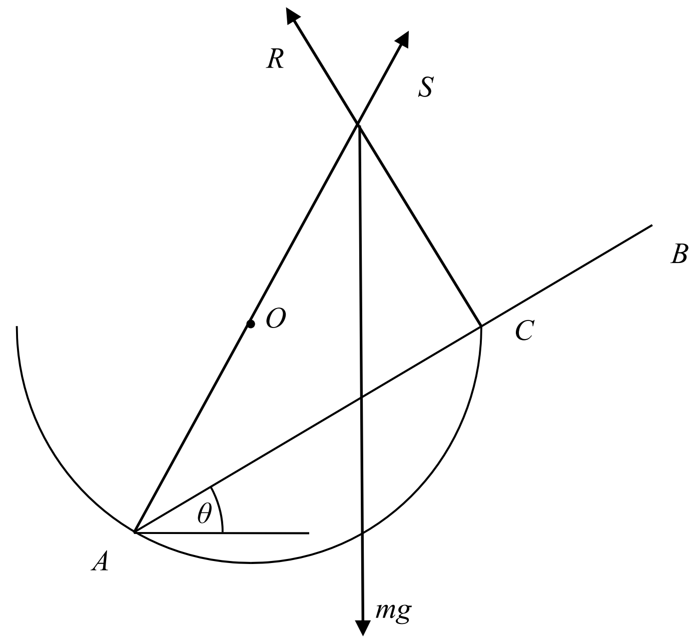
Q	Solution	Marks	Total	Comments
3	Force acting against gravity is $mg\sin\theta$ Force acting against gravity and resistance is $mg\sin\theta + 8000$ $= 1500 \times g \times \sin\theta + 8000$ $= 8588$ N or 8590 N Using power = force × velocity $= 8588 \times 22$ $= 188\,936$ W $= 189$ kW	 M1 A1 M1 dep A1 A1	 5	 Condone $\cos\theta$ or -1 for M marks Accept 188.9 or 188
	Total		**5**	
4(a)	Symmetry	E1	1	
(b)	Moments about AB: $300\sigma.15 + 100\sigma.5 + 300\sigma.15 = 700\sigma.\,x$ $x = \dfrac{9500}{700}$ $= \dfrac{95}{7}$ or 13.6 cm	 M1A1 A1	 3	 (condone lack of σ) M1 needs correct total marks
(c)	Distance from HG is 16.4 cm $\tan\theta = \dfrac{15}{16.42857}$ $= 0.913043$ $\theta = 42.3974^\circ$ $\theta = 42^\circ$	B1 M1 A1 A1	 4	 Seeing both 15,16.4 and tan [48° probably B1, M1] NB $\dfrac{13.6}{15}$ etc $\Rightarrow 42^\circ$ no marks
	Total		**8**	

Q	Solution	Marks	Total	Comments
5(a)	Using F = ma:			
	$-4v^{\frac{1}{3}} = 12\dfrac{dv}{dt}$	B1		
	$\therefore\ \dfrac{dv}{dt} = -\dfrac{1}{3}v^{\frac{1}{3}}$			
	$-3\int \dfrac{dv}{v^{\frac{1}{3}}} = \int dt$	M1		condone $-$, 3 incorrect side
	$-3 \times \dfrac{v^{\frac{2}{3}}}{\frac{2}{3}} = t + c$	A1		condone lack of $+ c$
	$-\dfrac{9}{2}v^{\frac{2}{3}} = t + c$			
	When $t = 0$, $v = 8 \Rightarrow c = -18$	M1A1		
	$-\dfrac{9}{2}v^{\frac{2}{3}} = t - 18$			
	$v^{\frac{2}{3}} = 4 - \dfrac{2}{9}t$			
	$v = \left(4 - \dfrac{2}{9}t\right)^{\frac{3}{2}}$	A1	6	
(b)	Particle is at rest when $4 - \dfrac{2}{9}t = 0$			
	The value of t is 18	B1	1	
	Total		**7**	
6(a)	Resolve vertically:			
	$T\cos\theta = mg$	M1		M1 for $T\cos\theta$ or $T\sin\theta$ and mg
	$34\cos\theta = 2 \times 9.8$	A1		
	$\cos\theta = \dfrac{19.6}{34}$			
	$\theta = 54.8°$	A1	3	
(b)	Resolve horizontally for particle:			
	$\dfrac{mv^2}{r} = T\sin\theta$	M1		M1 for $T\cos\theta$ or $T\sin\theta$
	$v^2 = \dfrac{34\sin 54.8 \times 0.8}{2}$	A1 ft from **(a)**		
	$v^2 = 11.113$			
	Speed is 3.33 m s^{-1}	A1	3	Accept 3.34
(c)	Time taken is $2\pi r / v$	M1		Or find ω and use $\dfrac{2\pi}{\omega}$
	$= 1.51$ sec	A1ft	2	
	Total		**8**	

Q	Solution	Marks	Total	Comments
7(a)	Using conservation of energy:			
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgh$	M1		for 3 terms, 2 KE and 1 PE
	$\frac{1}{2}\times 3\times v^2 = \frac{1}{2}\times 3\times 4^2 - 3\times g\times 1.2(1-\cos 25)$	M1 A1		M1A1 for finding h [M1 for $1.2(1-\cos 25\, or\ \sin 25)$]
	$v^2 = 4^2 - 2.4\times g(1-\cos 25)$			
	$v^2 = 16 - 2.2036$			
	$v = 3.71\,\text{m}\,\text{s}^{-1}$	A1	4	Accept 3.7, 3.70, 3.72
(b)	Resolving radially:			
	$T = mg\cos 25 + \frac{mv^2}{a}$	M1A1		M1 accept $\cos 25$ or $\sin 25$, + or $-$ sign and $\neq 2$
	$= 26.645 + 34.491$			A1 fully correct and substituted
	$= 61.1$ N	A1	3	Accept 61.0 or 61
	Total		**7**	

Q	Solution	Marks	Total	Comments
8(a)	Work done = $\int_0^e \frac{\lambda x}{l}\,dx$	M1		SC1 $\int_0^e \frac{\lambda e}{l}\,de$
	$= \left[\frac{\lambda x^2}{2l}\right]_0^e$	A1		SC1 $\int \frac{\lambda x}{l}\,dx$ with no limits
	$= \frac{\lambda e^2}{2l}$	A1	3	
(b)(i)	Using $T = \frac{\lambda x}{l}$:			
	$5g = \frac{392x}{1.6}$	M1		
	$x = \frac{5g \times 1.6}{392}$			
	$= 0.2$			
	Extension is 0.2 m	A1	2	
(ii)	When extension is 0.6 m, EPE $= \frac{\lambda x^2}{2l}$	B1		B1 for 0.6
	$= \frac{392 \times (0.6)^2}{2 \times 1.6}$	M1		
	$=$ 44.1 J	A1	3	
(iii)	Let y metres be distance particle is above A.			
	C of energy, when particle has speed 0.8 m s^{-1}, gives			
	$5 \times g \times y + \frac{392 \times (0.6 - y)^2}{2 \times 1.6} + \frac{1}{2} \times 5 \times 0.8^2$	M1A1		M1 4 terms, 2 correct M1A1 4 terms, 3 correct M1A2 4 terms correct
	$= \frac{392 \times (0.6)^2}{2 \times 1.6}$	A1F		Ft answer to **(b)(ii)**
	$49y + 122.5(0.6 - y)^2 + 1.6 = 122.5 \times 0.6^2$ $49y - 147y + 122.5y^2 + 1.6 = 0$ $122.5y^2 - 98y + 1.6 = 0$			
	$y = \frac{98 \pm \sqrt{98^2 - 4 \times 122.5 \times 1.6}}{2 \times 122.5}$			
	$y = \frac{98 \pm 93.9148}{245}$			
	$=$ 0.016674 and 0.7833	A1		if x used instead of $0.6 - y$, A1 here for $x = 0.5833...$
	Speed first becomes 0.8 when $y = 0.0167$	E1	5	
	Total		**13**	

Q	Solution	Marks	Total	Comments
9(a)	Smooth, hence reaction is perpendicular to possible movement	E1	1	
(b)				



Q	Solution	Marks	Total	Comments
		B2	2	B1 for 2 forces correct
(c)	Resolving along the rod:			Or geometrically:
	$S\cos\theta = mg\sin\theta$	M1A1		three forces act through a point B1 M1 is for 2 or 3 terms; 1 term correct (could be horizontal force at C used) $\left[\text{forces act through point } D\right]$
	Moment about C: $S\,2a\cos\theta\,.\sin\theta$			
	$= mg(2a\cos\theta - \tfrac{1}{2}l)\cos\theta$	M1A1		$AD\cos 2\theta = \dfrac{l}{2}\cos\theta$ M1A1
	$4a.S\sin\theta = mg(4a\cos\theta - l)$			$AD\cos\theta = 2a\cos\theta$ M1
	Dividing: $4a\tan\theta = \dfrac{4a\cos\theta - l}{\sin\theta}$ $l = 4a\cos\theta - 4a\sin\theta\tan\theta$			$l = \dfrac{4a\cos 2\theta}{\cos\theta}$ A1
	$l = \dfrac{4a\cos 2\theta}{\cos\theta}$	A1	5	

Q	Solution	Marks	Total	Comments
9 cont	**or** Resolving perpendicular to S: $R \cos\theta = mg \cos 2\theta$	(M1A1)		
	Moments about A: $R\, 2a\cos\theta = mg \tfrac{1}{2} l \cos\theta$	(M1A1)		
	$4a R = mgl$ $4amg \cos 2\theta = mgl \cos\theta$ $l = \dfrac{4a\cos 2\theta}{\cos\theta}$	(A1)		
	or Resolving horizontally: $R \sin\theta = S \cos 2\theta$ Resolving vertically: $R\cos\theta + S \sin 2\theta = mg$	(M1A1)		Both attempted for M1 Both correct for A1
	Moments about A: $R\, 2a\cos\theta = mg \tfrac{1}{2} l \cos\theta$	(M1A1)		
	$4a R = mgl$ $R\cos\theta + R\dfrac{\sin\theta}{\cos 2\theta} \sin 2\theta = 4a\dfrac{R}{l}$ $l = \dfrac{4a\cos 2\theta}{\cos\theta}$	(A1)		
	Total		**8**	
	TOTAL		**75**	